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Phil. Trans. R. Soc. Lond. A 1973 **274**, 279-284
doi: 10.1098/rsta.1973.0053

Phil. Trans. R. Soc. Lond. A. **274**, 279–284 (1973) [279]
Printed in Great Britain

Ultra sensitive laser interferometers and their application to problems of geophysical interest

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A 30 m laser strainmeter is currently being operated in an unworked gold mine near Boulder, Colorado. The strainmeter consists of an evacuated Fabry–Perot interferometer illuminated by a 3.39 μm He–Ne laser. A second 3.39 μm laser is stabilized by means of saturated absorption in methane and its wavelength serves as the reference length for the system. We shall describe the instrument in some detail and present the latest results in our investigation of the Earth tides and the Earth normal modes.

Introduction

We present here recent results obtained using the 30 m laser strainmeter located in a mine near Boulder, Colorado.

Description of the apparatus

Figure 1 shows a block diagram of the strainmeter and its associated electronics.

A 30 m evacuated Fabry–Perot interferometer is mounted on piers and is oriented along an axis approximately 7° west of north. The interferometer is 60 m below ground level.

The interferometer is illuminated by a 3.39 μm helium-neon laser. A servo loop piezoelectrically tunes the laser to keep its wavelength coincident with one of the transmission maxima of the long interferometer. The frequency of the laser is therefore related to the length of the interferometer by

$$f = nc/2L,$$

where n is an integer, c is the velocity of light and L is the length of the interferometer. Then

$$\Delta f/f = -\Delta L/L.$$

A second 3.39 μm laser is stabilized using saturated absorption in methane (Barger & Hall 1969; Levine & Hall 1972). The beat frequency between these two lasers is extracted and recorded for further processing. Thus

$$\Delta f_{\text{beat}}/f = \Delta L/L \quad \text{or} \quad \Delta f_{\text{beat}} = (8.85\times 10^{13})\ (\Delta L/L).$$

A more detailed description of the apparatus may be found in Levine & Hall (1972).

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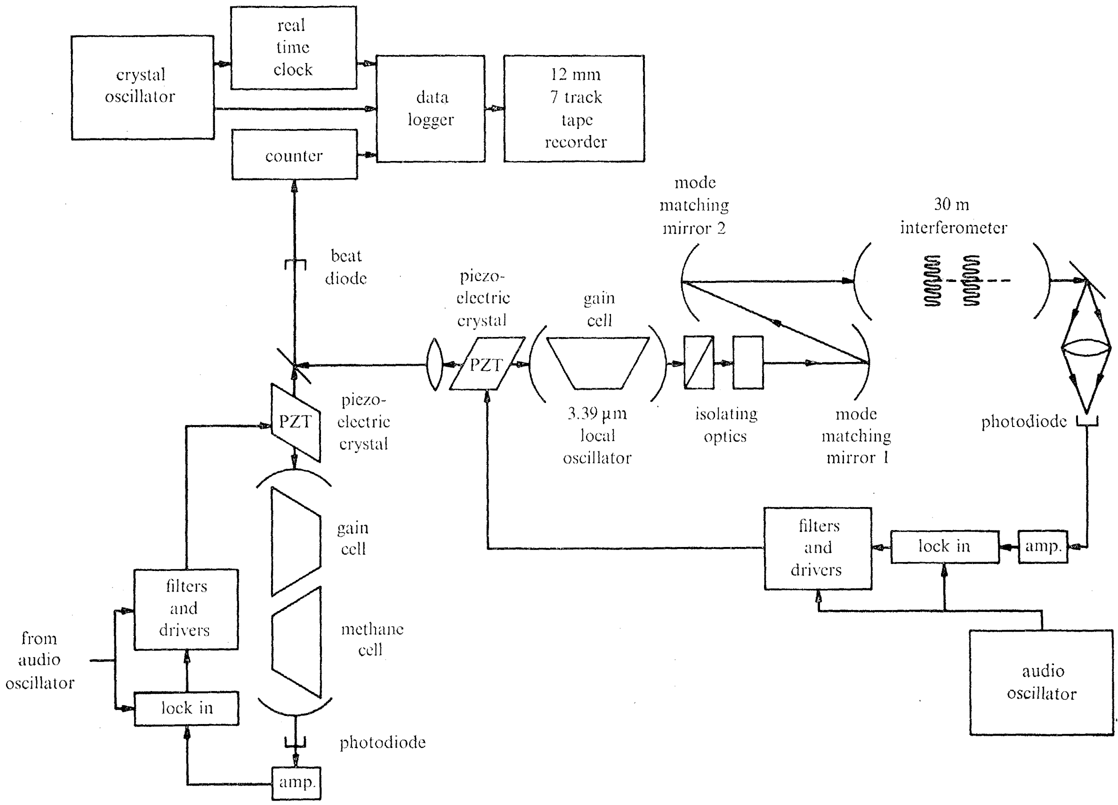

FIGURE 1. Block diagram of the 30 m laser strainmeter.

SENSITIVITY OF THE SYSTEM

The sensitivity of the system is limited by the noise generated in the servo electronics and by the fluctuations in the frequency of the methane stabilized laser. We characterize this noise at any frequency by the noise-equivalent strain amplitude, that is, by a fiducial strain generator in the Earth which would produce the noise actually observed at the output of the system.

The methane laser contribution to the noise equivalent strain amplitude has been measured in a separate series of experiments (Barger & Hall 1970; Levine & Hall 1972). The contribution of the long path servo lock to the system noise cannot be measured directly since it is not feasible to isolate the interferometer from the Earth. Rather the servo frequency noise is inferred from the measured signal-to-noise ratio in the detector and from the known characteristics of the electronics. In the vicinity of 1 Hz, for example, the total noise equivalent strain amplitude is $2 \times 10^{-13} (\Delta L/L)/\mathrm{Hz}^{\frac{1}{2}}$. In this vicinity the noise-equivalent strain amplitude goes approximately as the square root of the frequency and is several orders of magnitude less than the Earth noise.

The long path servo lock and the methane stabilizer pick the centre of emission features whose full widths at half maximum are on the order of 100 kHz. (The finesse of the long path is about 50.) Thus the sensitivity of the system is roughly equivalent to the sensitivity which would be obtained using a 1500 m long Michelson strainmeter and a reference length consisting of a 30 cm long passive Fabry–Perot cavity having a finesse of order 5000. Although a passive cavity with a finesse of 5000 is theoretically possible it is difficult to build.

Thus the strainmeter, although only 30 m long, has an inherent sensitivity at least as great as longer instruments in operation elsewhere (Berger & Lovberg 1970; Vali & Bostrom 1968). What it lacks is the ability to average over crustal inhomogeneities. Our experience suggests that, at least at our current site, this is not a serious limitation. In fact, although physical limitations would prevent us from constructing a much longer instrument at our current site, there is nothing in the design of the instrument which would prevent an increase in the length of the interferometer if a longer site were available.

Results

(a) *The normal modes*

We are interested in measuring the normal mode excitations generated by relatively large earthquakes. As a preliminary step in this measurement we have measured the power spectrum of the Earth noise during quiet periods. In figure 2 we show the noise power spectrum in the

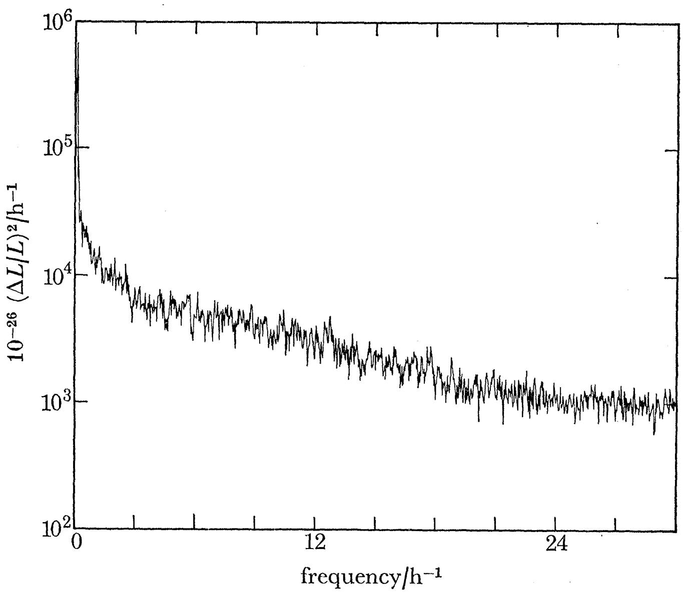

FIGURE 2. Power spectrum of the noise in the normal mode band. The data were filtered before transformation with a bandpass filter having essentially unity gain from 1 to 30 c/h and falling at 6 dB/octave below 1 c/h and above 30 c/h.

frequency range 1 to 30 c/h. The power estimate in each frequency band has the stability equivalent to a χ^2 distribution with 32 degrees of freedom. At 10 c/h the noise power is about 2×10^{-23} $(\Delta L/L)^2/\mathrm{h}^{-1}$. In contrast, Smith (1966) has reported normal mode power of about 3×10^{-22} $(\Delta L/L)^2/\mathrm{h}^{-1}$ following the Alaskan earthquake of 1964. We would therefore expect to be able to detect the mode excitations from a comparable event with a power signal-to-noise ratio exceeding 10 to 1.

(b) *The Earth tides*

We are also investigating the strain tides. In figure 3 we show a sample of the Earth tide record obtained with our strainmeter.

The data have been obtained by low pass filtering the beat frequency with a simple R–C network having a 3 dB corner at a period of 80 s and whose response falls at 6 dB/octave for periods less than 80 s.

On the same plot we have superimposed the theoretical tidal time series obtained from a tidal program provided by the Institute for Geophysics and Planetary Physics in San Diego, California (J. Berger 1970, personal communication).

There are several points worth noting: (1) the drift in the baseline of the measured tides is vanishingly small. Our best estimate is that it is less than 1×10^{-10} $(\Delta L/L)$ per day. (2) Although there is not a large obvious phase error between theory and experiment, the experimental tides are systematically too small in amplitude.

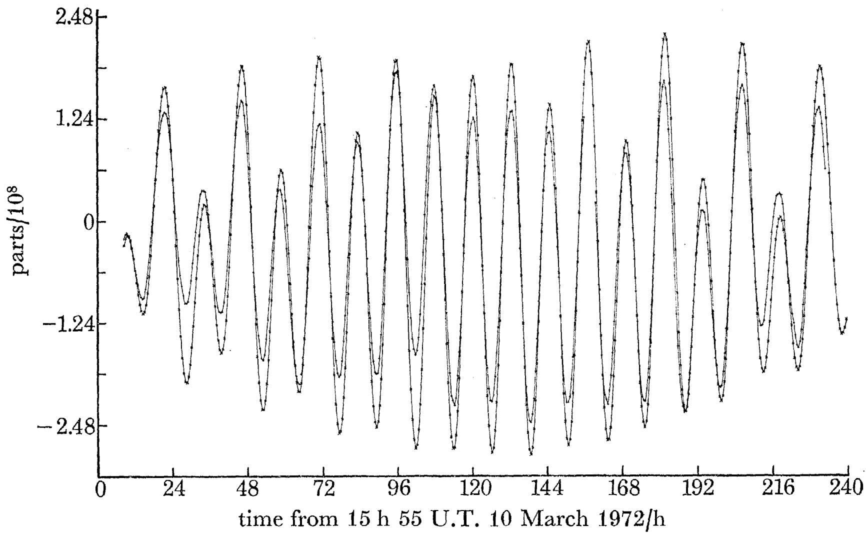

FIGURE 3. Experimental and theoretical strain tides as a function of time. The experimental data have been lowpass filtered with a filter whose transmission is unity from d.c. to a period of 80 s and whose transmission falls at 6 dB/octave for periods shorter than 80 s. No other filtering or detrending has been done. ×, theoretical; +, experimental.

In order to investigate these discrepancies in a more quantitative manner we have inserted several adjustable parameters into the theoretical tidal computation. We have then varied these parameters so as to minimize in a least-squares sense the discrepancies between theory and experiment. For technical reasons we have chosen to work in the time domain rather than to decompose the tides into their frequency components. We have chosen to vary the attenuation parameter, G, and the time delay τ. We have constructed a residual time series, R, whose ith element at time t_i is given by

$$R_i(t_i) = E(t_i) - G^* T(t_i + \tau),$$

where E and T are the measured and calculated tides respectively. We find that ΣR_i^2 is minimized in a least-squares sense when $G = 0.78 \pm 0.02$ and $\tau = +11.5 \pm 1.0$ min.

The estimates of the uncertainties in G and τ are determined assuming that the uncertainties are of a purely statistical nature. They have been obtained from the mean squared residuals using conventional least-squares methods.

In addition there is an uncertainty in the determination of G which comes from a possible long-term drift in the calibration of the frequency-to-voltage converter used in the low pass filter. This error, which is on the order of 5 %, tends to make G systematically too small. Although the sources of this problem have since been recognized and corrected, it was not possible to eliminate the uncertainty from measurements already completed. Thus the value of G quoted above is probably too small by about 5 %.

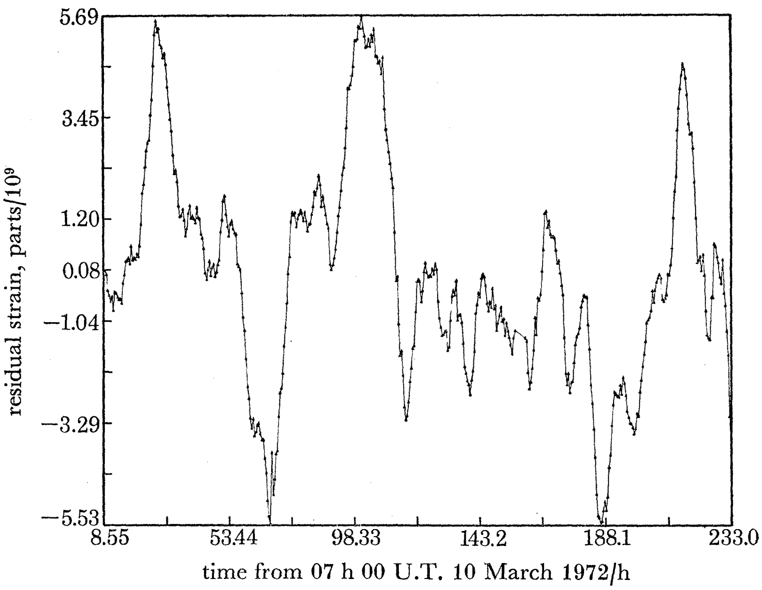

FIGURE 4. Tidal residuals, experimental tides less adjusted theoretical tides. The attenuation and time delay have been obtained from a least-squares analysis. The positive time delay (11.5 min) implies that the experimental tides lead the theoretical values.

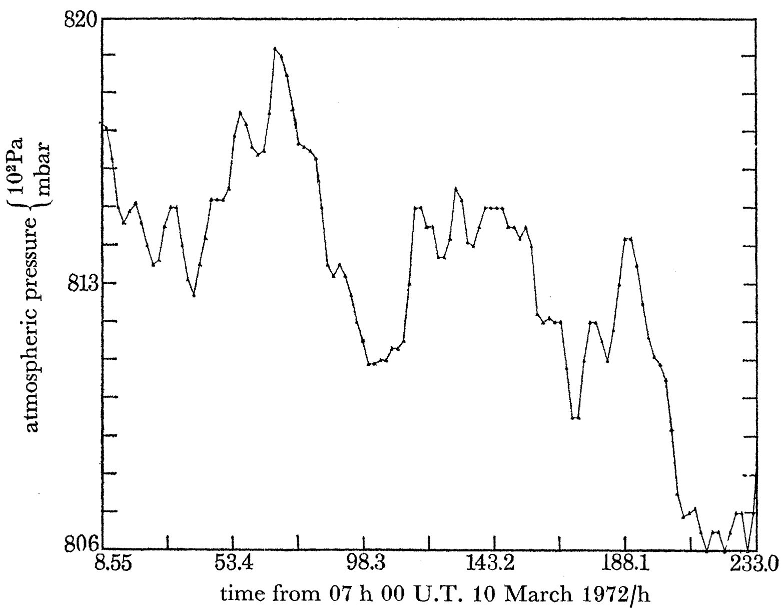

FIGURE 5. Atmospheric pressure fluctuations. The time period is the same as that in figure 4. Data provided courtesy of National Centre for Atmospheric Research, Boulder, Colorado, U.S.A.

In figure 4 we show the time series R obtained using the optimum values of G and τ obtained above. The residuals are still far from random suggesting that our simple model is not yet adequate.

In figure 5 we show the fluctuations in atmospheric pressure in Boulder as a function of time.† Figures 4 and 5 cover the same time period and a comparison of the two figures shows that all

† The barometric pressure data were provided through the courtesy of the National Center for Atmospheric Research, Boulder, Colorado, U.S.A.

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of the large peaks in the residuals correlate with fluctuations in atmospheric pressure. The correlation coefficient is obviously negative and both its sign and magnitude are in rough agreement with the results of a simple calculation of the expected effect on the local rock.

If that part of the residuals which is correlated with atmospheric pressure is removed, the adjusted residuals do not show any systematic behaviour and show random fluctuations of order 1×10^{-9} ($\Delta L/L$). We conclude that our three parameter fit (delay time, attenuation constant and atmospheric pressure correlation coefficient) is able to explain the observed strain signal with residuals on the order of 1×10^{-9} ($\Delta L/L$) or less.

It is not true, however, that by reducing the residuals to roughly the 5 % level the adjustable parameters are known to that accuracy. The three parameters are themselves correlated and a certain amount of trade-off among them is possible. Nevertheless, we feel confident that our model contains the essential physics of the problem.

Finally we ought to mention that the entire tidal analysis could have been done equally well in the frequency domain. In such an analysis we would decompose the theoretical tides into a sum of discrete frequency components. We would then adjust the amplitude and phase angle of each component to minimize the residuals.

Although both methods are, in general, equally efficient in fitting the data, the various tidal residuals are attributed to very different mechanisms in the two analyses. The time domain analysis attributes the residuals to attenuation and time delay, while in the frequency analysis we would speak of a frequency dependence of the local transfer function. Noise in the data and correlations between the adjustable parameters will render these two descriptions not exactly equivalent, although there is, of course, a direct connexion between the two in principle.

Conclusions

We feel that the data presented here establish beyond question that one can obtain reasonable results using strainmeters below ground and that if a suitable tunnel is available an instrument located in the tunnel is not *a priori* inferior to one located on the surface or in a trench.

We are, unfortunately, unable to unambiguously answer the question of whether crustal inhomogeneities place short strainmeters at a disadvantage relative to longer ones. An examination of tidal residuals might shed some light on the question, and one of our current efforts is to see if it is possible to give some theoretical justification to the empirically determined constants in our tidal theory.

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